

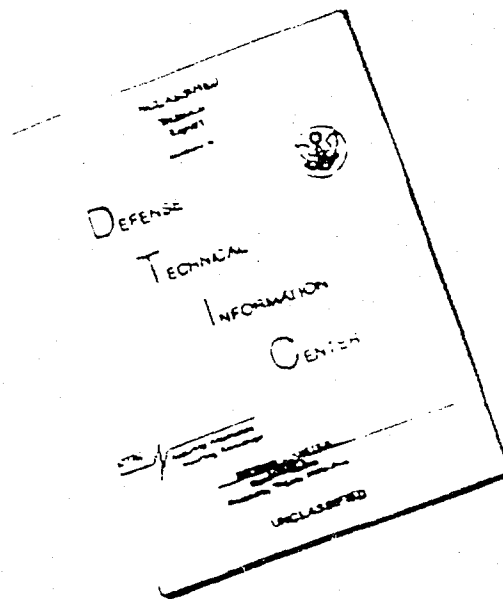
[illegible]

REC'D  
MAY 18 1977  
REGISTERED  
C  
H

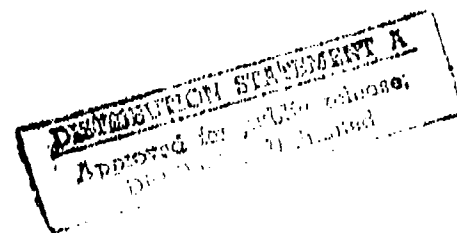
**DISTRIBUTION STATEMENT A**  
Approved for public release,  
Distribution Unlimited

12

# DISCLAIMER NOTICE



THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.



WAVE MOTION IN A THICK-WALLED  
FLUID-FILLED HOSE

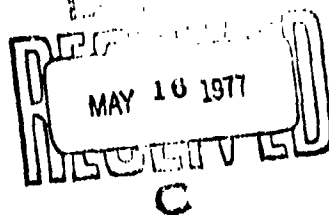
The research documented in this Final Report was  
sponsored under Contract N00014-75-C-0633 by:

Office of Naval Research  
Department of the Navy  
Washington, D. C. 22217

Contract Authority NR 386-905/1-8-75 (220)

Reproduction in whole or in part is permitted for  
any purpose of the United States Government.

Prepared by: Dynell Electronics Corporation  
Melville, New York 11746



DY-764R

April 1977

UNCLASSIFIED

Security Classification

## DOCUMENT CONTROL DATA - R &amp; D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Dynell Electronics Corporation ✓ 75 Maxess Road Melville, New York		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
2b. GROUP Not Applicable			
3. REPORT TITLE ⑥ Wave Motion in a Thick-Walled Fluid-Filled Hose.			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) ⑨ Final Report, 75 JAN 01 - 77 APR 30			
5. AUTHOR(S) (First name, middle initial, last name) ⑩ Oscar A./Lindemann		⑪ 1 Jan 75 - 30 Apr 77	
6. REPORT DATE ⑪ Apr 77		7a. TOTAL NO. OF PAGES 11	7b. NO. OF REFS none
8. CONTRACT OR GRANT NO. ⑬ NO. 014-75-C-0633 New		9a. ORIGINATOR'S REPORT NUMBER(S) ⑭ DY-764R ✓	
9b. PROJECT NO. c. ⑬ 10 45p. d.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) none	
10. DISTRIBUTION STATEMENT Distribution of this document is unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Office of Naval Research Department of the Navy Arlington, Virginia 22217	
13. ABSTRACT The subject of this report is the analytical solution for wave motions, in the fundamental mode, of an elastic tube filled with fluid when the wall thickness is not constrained to be small.  The results obtained show the existence of two waves which travel at different speeds. Each of the two waves causes both longitudinal and radial displacements of both the tube wall and the fluid, but in different proportions.  The slower wave, in the limiting case of the thin-walled tube, is identified as the bulge wave. It is characterized by relatively large motions of the fluid, which contains most of the kinetic energy. Thicker walls produce a higher bulge-wave speed, but the inclusion of longitudinal stiffening elements has very little effect.  The faster wave is what has been called the "wall wave." This is mostly a longitudinal vibration of the hose material, with a relatively small participation by the fluid, and is greatly influenced by longitudinal reinforcement of the tube wall.			

DD FORM 1473  
1 NOV 65UNCLASSIFIED  
Security Classification

404 987

LR

14

KEY WORDS

LINK A

LINK B

LINK C

ROLE

WT

ROLE

WT

ROLE

WT

Wave Motion In A Thick-Walled Fluid-Filled Hose

Acoustic Waves, Turbulent Boundary Layer,  
Mathematical Model, Bulge Wave, Wall Wave,  
Longitudinal and Transverse Wave, Hose  
Stiffening, Wave Speed

NTIS  
D-8  
UNANNOUNCED  
NOTIFICATION

White Section ☒  
Bull Section ☐

DISTRIBUTION AVAILABILITY CODES  
Dist. AVAIL. and or SPECIAL

A

## WAVE MOTION IN A THICK-WALLED FLUID-FILLED HOSE

### INTRODUCTION

Random pressures on the outside surface of a fluid-filled hose, originating from a turbulent boundary layer or other sources, excite waves which travel along the fluid-filled hose as in a wave guide. Consequently, inside the hose there exist noise pressures which are correlated over large distances. An important characteristic of this phenomenon is the speed of such waves, which is in general well below the acoustic speed because of the relatively soft boundary which the hose presents to the inner column of fluid.

Previous mathematical models of such flexible hoses have reduced them to membranes with circumferential stresses only. Longitudinal motions and stresses in the wall were ignored. This model proved very adequate for hoses with thin walls.

Recent interest in hoses of smaller size, where hose material occupies a considerable part of the total cross section, requires that the analysis be extended to include the wave motion in the wall as well as the fluid.

Thus the subject of this report is the analytical solution for wave motions, in the fundamental mode, of an elastic tube filled with fluid when the wall thickness is not constrained to be small (Fig. 1).

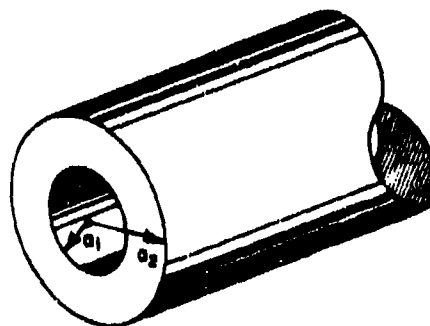


FIG. 1 MODEL OF THICK-WALLED HOSE

The present model then includes as limiting cases the previous thin-walled tube, where the ratio  $a_1/a_2$  of the radii is almost one, and the solid rod, where this ratio is zero. It also serves as a basis for studies of more complicated models where the inner fluid is replaced by a viscous or semisolid material.

In accordance with the results of previous investigations the outer fluid, the ocean in which the tube is submerged, is ignored, as wave propagation in the hose is not appreciably influenced by its presence.

The material of the tube is assumed to be homogeneous and isotropic but in a subsequent modification longitudinal reinforcing elements of an arbitrary elastic modulus have been included. The compressibility of both the wall material and the fluid is neglected only in the simplified final formulas which are valid for waves of lengths much greater than the tube's transverse dimensions; in these cases the effects of compressibility become negligible.

The outstanding result is the existence of two waves which travel at different speeds. Each of the two waves causes both longitudinal and radial displacements of both the tube wall and the fluid, but in different proportions.

The slower wave, in the limiting case of the thin-walled tube, is identified as the bulge wave. It is characterized by relatively large motions of the fluid, which contains most of the kinetic energy. Thicker walls produce a higher bulge-wave speed, but the inclusion of longitudinal stiffening elements has very little effect.

The faster wave is what has been called the "wall wave." This is mostly a longitudinal vibration of the hose material, with a relatively small participation by the fluid, and is greatly influenced by longitudinal reinforcement of the tube wall.

The system functions as a wave-vector filter, that is, there is an acceptance function defined as the ratio of inner-fluid pressure to outer forcing

pressure as a function of wave number, with frequency as a parameter. The acceptance function will show two peaks, or windows, at the wave numbers corresponding to the fast and slow waves. There is also a zero in the acceptance function, at the wave number corresponding to the speed of longitudinal plate waves in the hose material, and independent of the wall thickness and the inner fluid's density.

#### METHOD OF SOLUTION

The very extended mathematical development of the problem will be included in this report. The method followed was completely orthodox.

The wave equation in the tube wall is that corresponding to an isotropic elastic material. Both longitudinal and transverse components (having respectively a scalar and a vector potential) are present, and each has two terms described by modified Bessel functions. Thus four arbitrary constants are necessary for the general solution. In the fluid there is only one component and a fifth arbitrary constant must be introduced.

The determination of the five constants follows from the setting up of five equations describing boundary conditions. These boundary conditions are:

- o The radial displacements of the fluid and tube wall must coincide at the interface.
- o The tangential stress of the tube material must be zero at the inner surface.
- o The tangential stress must also be zero at the outer surface, in the absence of reinforcing material; when there is reinforcing material, this stress must be proportional to the longitudinal strain.
- o The normal (radial) stress in the tube material must balance the pressure of the inner fluid at the interface.



- o The normal (radial) stress in the tube material must balance the outer forcing pressure at the outer surface.

All these equations are transcendental because of the presence of the Bessel functions. When, however, the assumption is made that the wavelengths involved are long compared with the tube's outer radius, the arguments of the Bessel functions become small, and they can be approximated by their dominant terms. Thus the five constants became relatively simple functions of wavenumber. When there is no forcing pressure, the system of equations becomes homogeneous, and there can be solutions only for the values of wave speed that negate the system determinant. This results in a second-degree equation whose roots are the speeds of the slow and fast waves.

## RESULTS

In calculating and plotting the results, the wave speed is shown to depend on the tube's wall thickness. In addition, and as parameters, first the ratio of wall to fluid densities and then the influence of wall reinforcement are considered.

In the first case, when there is no wall reinforcement but the ratio of wall to fluid densities is allowed to vary, the wave speed for both waves is given as the solution of the following quadratic:

$$\eta^2 - [3 + \gamma - (\gamma - 1)\beta^2] \eta + 3\gamma(1 - \beta^2) = 0, \quad (1)$$

where the symbols have the following meanings:

$\eta$  is a number proportional to the square of the wave speed,  $\eta = \rho c^2 / \mu$ , where  $\rho$  is the density of the hose material,  $\mu$  its shear modulus and  $c$  the wave speed. The definition results in  $\eta$  being 1 for plane shear waves in the material of the hose when it has no bounds,  $\eta = 3$  for longitudinal waves in a free bar of the material, and  $\eta = 4$  for those in a plate of the material.

$\gamma = \rho/\rho_0$ , the ratio of the densities of the hose material and the fluid.

$\beta$ , the ratio of the inner to the outer radius of the hose.

A plot of  $\eta$  as a function of  $\beta$ , with  $\gamma$  as parameter, is shown in Fig. 2.

The curves where  $\eta \leq 3$  correspond to the slow wave. The slopes of the curves at  $\eta = 0$  give as the limiting speed for a thin-walled tube  $c = \sqrt{3\mu h/2\rho_0 a}$  where  $h$  is wall thickness and  $a$  the outer radius. This shows that, in the present case of incompressible material, the "circumferential modulus"  $E$  used previously in calculating the bulge-wave speed is to be defined as the ordinary Young's modulus for a bar,  $E = 3\mu$  for incompressible material.

The fast wave has  $\eta \geq 3$ , the lower value being that for a solid rod, and the higher values showing the remarkable influence of the fluid fill. For a thin-walled tube,  $\eta = 4$ , showing that the tube vibrates longitudinally over a fluid column that acts as a stiff core.

The second case studied is where there is an outer sheath of thin, flexible but longitudinally stiff reinforcing material. The densities of the wall and the fluid are supposed equal throughout. The wave speeds are now given by:

$$\eta^2 - 2(1 + \nu)\eta + (2\nu + 1)(1 - \beta^2) = 0, \quad (2)$$

where,

$$\nu = 1 + \frac{Mb}{\mu} \frac{a_2^2}{a_2^2 - a_1^2},$$

$M$  is the longitudinal elastic modulus of the sheath, and  $b$  its thickness (or the equivalent thickness of the reinforcing fibers). Where there is no reinforcement,

$\nu = 1$ , which is the case previously studied, when the reinforcement becomes infinitely stiff,  $\nu \rightarrow \infty$ .

The slow wave is very insensitive to the value of  $\nu$ , so there is only a small difference between the values of  $\eta$  when  $\nu = 1$  (as in Fig. 2) and when  $\nu = \infty$ ; then  $\eta$  is given by

$$\eta = \gamma (1 - \beta^2).$$

For a thin-walled tube this gives

$$c = \sqrt{\frac{2\mu h}{\rho_0 a}},$$

i.e., the reinforcement tends to increase the wave speed to the value calculated with  $E = 4\mu$ , the plate modulus.

The fast wave is made faster by the reinforcement, the asymptotic value of  $\eta$  as  $\nu \rightarrow \infty$  being  $2\nu + 1 + \beta^2$ . This means that the shape of the curve of  $\eta$  versus  $\beta$  hardly changes shape as  $\nu$  increases. Fig. 3 shows the curves for  $\nu = 1$  and  $\nu = 100$ .

#### TRANSFER FUNCTION

The free-going waves given by the values of  $\eta$  in figs. 2 and 3 correspond to the condition where there is pressure in the fluid with no outside forcing pressure. They are the poles of the transfer function  $p/p_0$  defined as the ratio of inside pressure over outside pressure.

There is also a zero in the transfer function, at the value of  $\eta$  where an outside pressure produces no corresponding inner pressure. This value of  $\eta$ , called  $\eta_0$ , is independent of  $\beta$  and  $\gamma$ , and is obtained in the solution of the system of five equations as

$$\eta_0 = 2(1 + \nu).$$

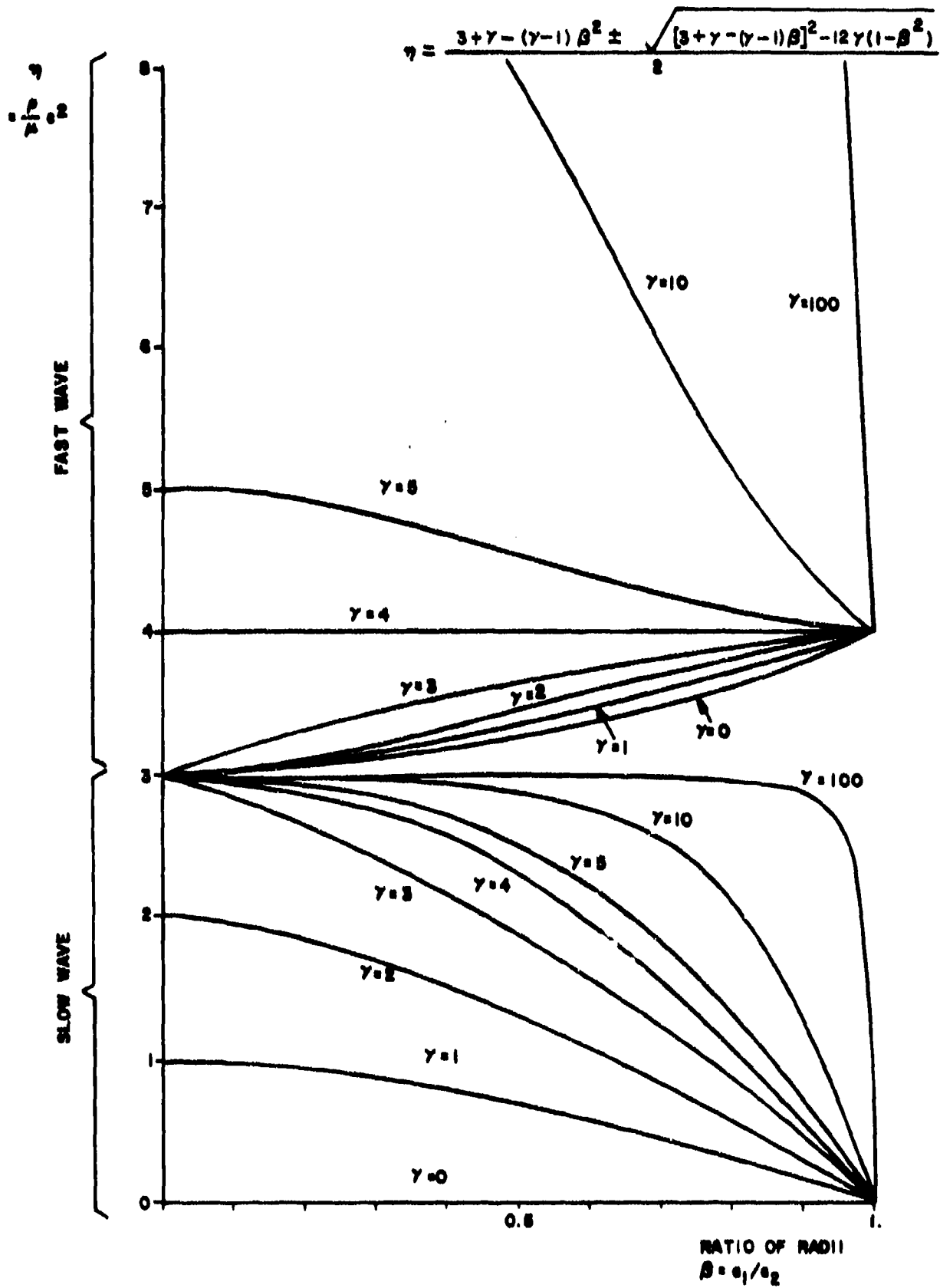


FIG. 2. WAVE SPEED FOR HOMOGENEOUS HOSE MATERIAL

$$\eta = 1 + \nu \pm \sqrt{(1 + \nu)^2 - (2\nu + 1)(1 - \beta^2)}$$

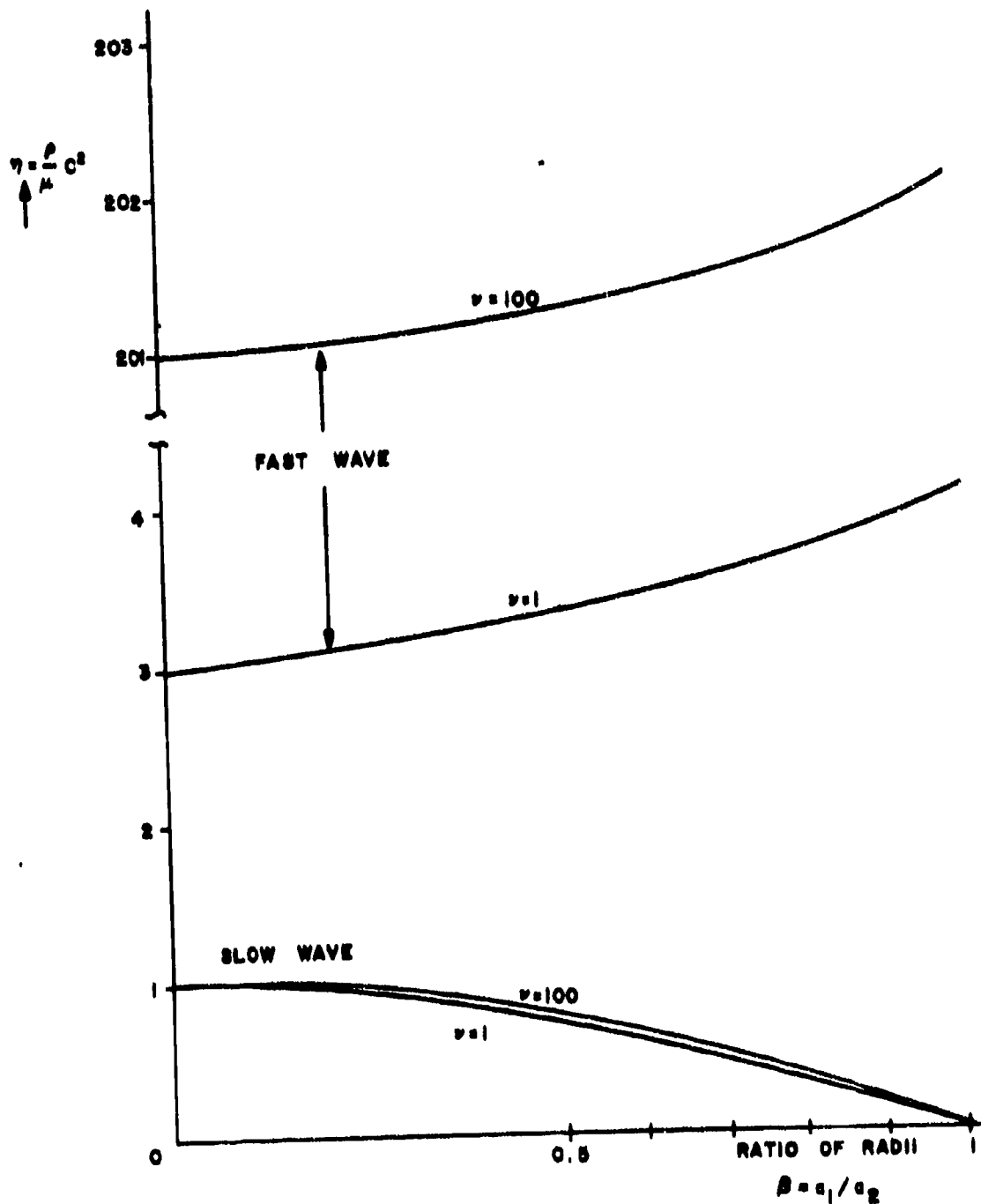


FIG. 3 WAVE SPEED AS INFLUENCED BY REINFORCEMENT

The transfer function is given in general by

$$\frac{p}{p_0} = \frac{\eta(\eta - \eta_0)}{(\eta - \eta_1)(\eta - \eta_2)},$$

where  $\eta_1$  and  $\eta_2$  are the values of  $\eta$  for slow and fast waves respectively.

Then for the first case studied, where  $\nu = 1$  and  $\gamma$  is arbitrary, we use Eq. (1) for the denominator and we have

$$\frac{p}{p_0} = \frac{\eta(\eta - 4)}{\eta^2 - [3 + \gamma - (\gamma - 1)\beta^2]\eta + 3\gamma(1 - \beta^2)};$$

in the second case studied, where  $\gamma = 1$  and  $\nu$  is arbitrary, we use Eq. (2):

$$\frac{p}{p_0} = \frac{\eta(\eta - 2 - 2\nu)}{\eta^2 - 2(1 + \nu)\eta + (2\nu + 1)(1 - \beta^2)}.$$

Fig. 4 graphs this last case for  $\nu = 1$  (no reinforcement) and  $\nu = 100$  (heavy reinforcement), against the nondimensional wavenumber  $1/\sqrt{\eta} = (k_2/\omega)\sqrt{\mu/\rho}$ . The figure shows that over a region of wavenumbers at and below  $1/\sqrt{\eta} = 1$  the inner pressure  $p$  is larger than the outer pressure  $p_0$ ; this is the "window" of the system, the acceptance region of the wavenumber filter, where noise pickup is enhanced.

The equations in this report make it possible to locate and define this "window" for a wide range of hose configurations, and are expected to be therefore useful in predicting noise levels.

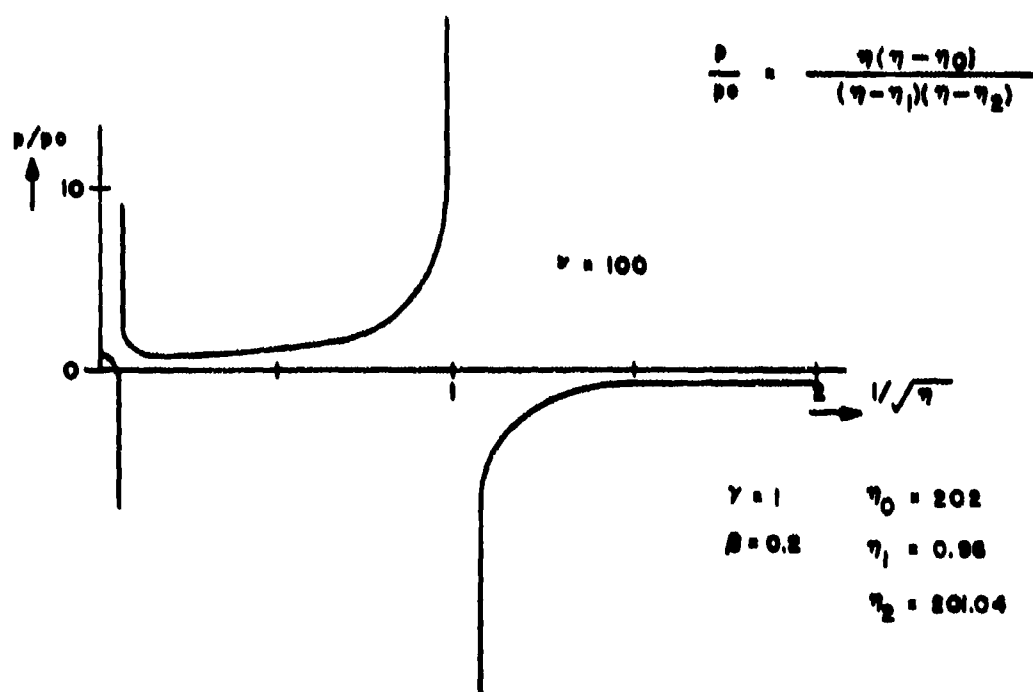
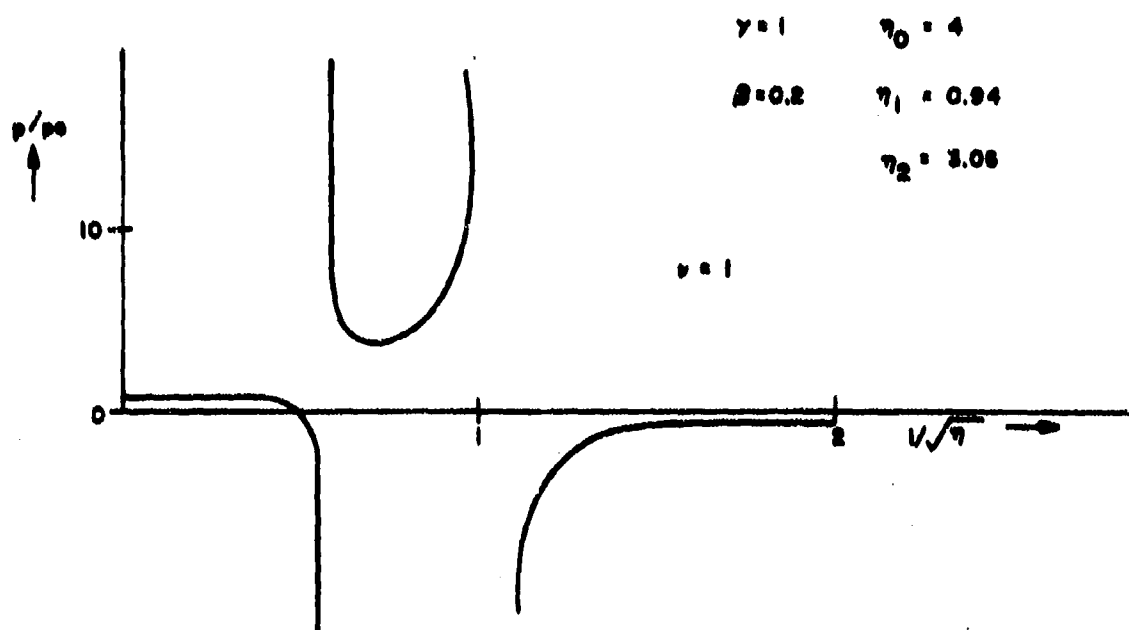


FIG. 4 TRANSFER FUNCTION AS INFLUENCED BY REINFORCEMENT

## CONCLUSIONS

This investigation has clarified completely the role the mechanical properties of the wall play in the determination of wave speeds in fluid-filled hoses. Previous models merely defined the wall as an elastic boundary for the fluid, and assigned to it suitable properties.

The results for  $\gamma$  not equal to unity are of theoretical value only at present, since the densities of the hose and the fluid are normally close to that of water for present materials, but aid in understanding wave progression in the system.

The ability to predict wave motion in thick-walled tubes is the main outcome of this study. It will serve as a sound basis for more elaborate models which are now being studied (viscous fluid fill, solid fill, etc.). Finally, the effect of restricting the longitudinal motions of the wall by stiffening members, necessary in any practical model, has been adequately taken care of and assessed.